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ABSTRACT

Analytical exploratory investigations indicated that transition effects such as streaming will cause a considerable spatial variation in the neutron spectra across resonances; streaming leads to opposite effects in the forward and backward directions. The neglect of this spatial and angular variation of the transitory resonance spectra is an approximation that is common to all current methodologies.

An integral transport theory formalism was developed for the description of spatially dependent spectra in isolated resonances. This treatment differentiates between forward and backward directed components of the neutron flux in slab geometry.

This theory was applied to an isolated actinide resonance in a simplified fast reactor blanket problem. The resonance spectra of the directional flux components , ϕ^+ and ϕ^-, and even more so the 90° cone components, were shown to deviate significantly from the infinite medium approximation with the differences increasing with penetration. The changes in ϕ^+ lead to a decreasing scattering group constant which enhances neutron transmission; the changes in ϕ^- lead to an increasing group constant inhibiting backward scattering. Therefore, the changes in the forward and backward directed spectra both lead to increased neutron transmission.

Conversely, the flux ($\phi = \phi^+ + \phi^-$) was shown both in the analytical formulas and in the numerical solution to agree closely with the infinite medium approximation; the directional effects cancel in the summation. Therefore, flux-weighted ("diffusion theory") group constants cannot yield the required increase in transmission even using transport theory.

The forward and backward directed flux components were used as weighting spectra to illustrate the group constant changes for a single resonance. Results indicate these changes have a magnitude which can likely account for calculational underpredictions in the blanket region.

I. INTRODUCTION

The refinement of blanket physics predictions requires an understanding of the nature of the blanket transmission problem. Blanket physics predictions are considerably more complicated than core predictions because of the transitory nature of the neutron flux and spectra within the blanket. Most of the neutrons enter the blanket from the core; the intrinsic source is comparatively small. Thus, the spectrum softens considerably with increasing penetration as scattering slows down the transmitted neutrons. In addition, the flux decreases significantly across the blanket and becomes increasingly forward biased. Therefore, the separability in space and energy which is assumed in group constant generation is violated in the blanket region.

The blanket transmission problem can be considered a subset of what is commonly called the "deep penetration" problem which is generally understood as the prediction of the neutron flux as it leaves a source and penetrates large distances in a scattering/absorbing medium. Significant underpredictions of the neutron flux that increase with penetration have been consistently observed using standard methods. This problem occurs in many shielding and related damage calculations.

There are several basic assumptions which are made in all current group constant generation methods. The within-group weighting spectra are either simple expressions (e.g. 1/E or fission spectra) or they result from zero-dimensional calculations (e.g. MC^2) or a "narrow" resonance infinite medium approximation. No consideration is given to the possible use of special transitional (i.e. non-asymptotic) weighting spectra. Errors in the generation of group constants will obviously be reflected in flux predictions utilizing such data.

This paper focuses on the calculation of transitional spectra in the presence of resonance materials and the effect of these transitional resonance spectra on multigroup flux predictions in the blanket region. The blanket is commonly the first region for the radial transmission of the neutron flux coming from a fast reactor core. The radial reflectors and outer regions of a fast reactor will also be regions with a transitory flux. Since the flux transition is even more severe for these outer regions due to the total absence of an intrinsic neutron source, any discrepancies found in the blanket are expected to increase in magnitude for these outer regions. The blanket investigations described here reveal the onset of the deep penetration underpredictions.

Discrepancies between predictions and measurements have been fairly accurately quantified in the blanket region of the Purdue Fast Breeder Blanket Facility[1] (FBBF). The onset of the deep penetration underpredictions is observed as a C/E drop-off from 1.0 to 0.8 in the U-238 capture rate across a 51 cm blanket using both transport and diffusion theory with refined group constants. The cause of these blanket discrepancies is investigated and a mechanism for improving blanket predictions is indicated in this paper.

It appears that significant changes in the resonance spectra develop during neutron transmission. These transitional resonance spectra will be contrasted to spectral approximations used in current group constant generation methods in Section II. This comparison indicates a probable cause for the blanket underpredictions. Section II focuses on an understanding of the physical phenomena with the detailed treatment developed later.

In Section III, a model will be formulated to analyze the space and energy dependence of resonance spectra using integral transport theory. This technique will be applied to a single isolated U-238 resonance in Section IV; the results will be discussed in detail and compared to the standard group constant approximations. The need for "direction dependent group constants" will be shown. A preliminary analysis of the effect of transitional resonance spectra on the generation of group constants will be presented in Section V.

II. INFINITE MEDIUM AND TRANSITORY RESONANCE SPECTRA

The neglect of transitional effects in resonance self-shielding is a major approximation common to all current group constant generation methods. Generally, infinite medium spectra are used which lead to a space independent source. As a further simplification, the narrow resonance (NR) approximation is applied; all neutrons are assumed to scatter into a resonance from well above the resonance energy which leads to a source that is constant in energy across the entire resonance. These assumptions give the NR approximation of the resonance spectrum:

$$\phi(x,E) \approx \frac{\sigma_p}{\sigma_t(E)} \cdot \frac{\psi_0}{E} = f^{NR}(E)\frac{\psi_0}{E} \ , \tag{1}$$

where ψ_0 is the flux per unit lethargy between resonances, σ_p is the potential cross section and $\sigma_t(E)$ is the total cross section per resonance absorber nucleus, and $f^{NR}(E)$ is the self-shielding factor in the NR approximation.

Several methods calculate the resonance spectra more accurately than the NR approximation. This is particularly needed in some thermal reactor applications where resonances at epithermal energies are important. For these epithermal resonances, the energy loss per scattering on actinides is comparable with or even smaller than the resonance width and the resonances can no longer be considered "narrow." Most of these methods maintain an infinite medium treatment and strive for a better treatment of the energy dependence of the neutron source. An infinite medium scattering source is then used, leading to a flux shape of the form:

$$\phi(x,E) \approx \int_E^{\infty} \sigma_s(E' \rightarrow E)\phi(E')dE' / \sigma_t(E) \ ; \tag{2}$$

where $\phi(E')$ is the infinite medium spectrum. The MC^2–2 code[2] uses this treatment for the broad scattering resonances; the integral in the numerator is approximated by a very fine group zero-dimensional flux solution.

In a transition region, both the approximations presented in Eqs. (1) and (2) are inaccurate. The flux transition leads to a spatial dependency of the scattering source. Once the source is spatially non-constant, the differing attenuation rates within the resonance become important. As an example, consider the simple attenuation of the flux from a neutron source. The neutrons at the resonance energy will attenuate quickly; the neutrons in the interference dip will penetrate much further ("streaming"). However, the effect of these differing attenuation rates is described by a single group constant for most resonances. The typical error in this approach can be illustrated by the following simple comparison of linear attenuation formulas:

$$\frac{\phi_g}{2}[e^{-\Sigma^D x} + e^{-\Sigma^P x}] \neq \phi_g e^{-\Sigma_g x} \ , \tag{3}$$

where Σ^P is the cross section at the resonance peak, Σ^D is the cross section at the resonance dip, ϕ_g is the group flux, and Σ_g is an average group cross section. Representation of the entire resonance by a single cross section will obviously underpredict the streaming in the interference dips and overpredict the transmission in the resonance peak. The transmission in the interference dip will dominate asymptotically, but it is absent on the right hand side of Eq. (3) leading to an underprediction of the group flux at large distances.

In a blanket region, the flux, and therefore the source, is decreasing. Thus, by observation of Eq. (3) the flux peak corresponding to the interference dip will attenuate at a much slower rate than the flux dip. This will lead to a "tilting" of the resonance spectra with an increasing portion of the spectra concentrating in the interference dips with increasing penetration. Therefore, the flux transition leads to a spatial dependency of the spectral deformation as well as streaming effects. If these transitional spectra are used for group constant generation, the low cross sections receive increasing weight as the source decreases (e.g. with increasing distance from the core).

The simple illustration of the spatial effects in Eq. (3) is refined and further investigated by a more explicit treatment of the attenuation in a transition problem (see Ref. 3). In this treatment the source is considered to be a function of space, and the NR approximation is replaced by an attenuation formula applied to an energy independent source. Only the forward ($\mu = 1$) and backward ($\mu = -1$) angular fluxes (ϕ_+ and ϕ_-) are evaluated for simplicity.

$$\phi_+(x,E) = \tfrac{1}{2}\int_0^\infty S_t(x - l)e^{-\Sigma_t(E)l}dl \qquad \phi_-(x,E) = \tfrac{1}{2}\int_0^\infty S_t(x + l)e^{-\Sigma_t(E)l}dl \,. \tag{4}$$

A Taylor expansion is applied to the source at x so that the spatial integrations can be readily carried out. The result is:

$$\phi_+(x,E) = \frac{1}{2\Sigma_t(E)}\left[S_0(x) - \frac{S_1(x)}{\Sigma_t(E)} + \frac{S_2(x)}{\Sigma_t(E)^2} - \cdots\right] \,, \tag{5a}$$

$$\phi_-(x,E) = \frac{1}{2\Sigma_t(E)}\left[S_0(x) + \frac{S_1(x)}{\Sigma_t(E)} + \frac{S_2(x)}{\Sigma_t(E)^2} + \cdots\right] \,, \tag{5b}$$

where $S_1(x)$ and $S_2(x)$ are the first two derivatives of the spatial source.

One observes the expected streaming and spectral effects in the forward flux of Eq. (5). If S_1 is negative ($S(x)$ decreasing for increasing x) the second term in Eq. (5a) adds to the flux particularly in the resonance dip, where $\Sigma_t(E)$ is small, but nearly nothing at the resonance peak, where $\Sigma_t(E)$ is very large. Thus, the spatial variation of the source leads to a higher flux in the interference dips caused by the slower attenuation rate. However, the opposite effects are seen in the backward flux ϕ_-; here the $S_1(x)$ term is negative with a larger subtraction applied within the resonance dip since $\Sigma_t(E)$ is small. Thus, if $S_1(x)$ is negative, ϕ_+ is greater than ϕ_- for all energies with the differences much more pronounced at low cross sections.

The streaming effects lead to a forward angular flux bias ($\phi_+ > \phi_-$) with increasing blanket penetration. In addition, the flux tilting requires a decrease in the effective group constant because of the increasing importance of the low cross sections in the forward direction. Therefore, this transitional resonance effect should allow more neutron transmission, yielding blanket flux (and reaction rate) predictions more in agreement with experimental results.

Equation (2) gives the resonance spectrum with the energy dependence of the source taken into account; Eqs. (5) show the forward and backward spectra with the space dependence of the source taken into account. One would expect even greater effects when both dependencies are considered. In this case there will be a coupling effect between the resonance peak and interference dip. Neutrons scattering in the resonance peak (a highly probable reaction) can readily scatter into the near-by interference dip with little energy loss (and correspondingly little angular deflection). Thus, one would expect the resonance scattering to further enhance the spatial "build-up" of the interference flux peak in the forward direction. This coupling effect can only be analyzed by a detailed space-energy-angular treatment of the resonance spectra.

In summary, both the space dependence and energy dependence of the scattering source are needed to model the transitional effects on resonance spectra. An adequate treatment must address the space, energy, and angular variations of the transitional resonance spectra. Such a model for calculating the flux as a function of space and energy for a single resonance in a transition region is developed in the next section. Since transitional effects are subsequently shown to be important, the desirable application is to generate a new set of resonance weighting spectra which are used to replace the NR spectra in group constant generation.

III. GENERAL FORMULATION OF THE MODEL

Integral transport theory was chosen for this analysis because it allows an exact representation of the spatial attenuation (streaming) as well as a continuous energy representation of the sharply varying resonance cross section. Although the formulation of integral transport theory is based upon a spatial integration of the last collision, direct numerical solution of this integral equation accounts for all of the neutron collisions within the resonance. This accounting is realized by "stepping-down" in energy. The integral transport analysis is performed for the upper energies of the resonance first. For lower energies , neutrons scattered from the upper energies are part of the source. Therefore, the detailed energy and angular dependence of the neutron source is accounted for in this model.

The integral transport analysis is not practical for the entire flux solution (all spatial and energy detail). However, the goal of this model is a replacement of the NR approximation for individual resonances. Thus, a simplification is introduced by looking at isolated resonances. This limits the problem to a small energy range allowing the practical application of integral transport theory. Furthermore, as the eventual goal of this transport analysis is the generation of refined group constants, the spatial model may also be simplified to slab geometry. The resulting group constants can subsequently be applied to more complicated or higher dimensional geometries. In this approach, the zero-dimensional or infinite medium weighting spectra are replaced by transitional resonance spectra which depend on space and direction.

IIIA Theory

The starting point for this derivation is the integral transport equation for the angular flux in a one-dimensional slab. In the resonance energy range the fission and independent sources are negligible; thus, the only source is the scattering source:

$$q(x', E, \mu) = \int_{\Omega'} \int_{E'} \Sigma_s(E' \rightarrow E, \mu_s)\phi(x', E', \vec{\Omega}')dE'd\Omega', \tag{6}$$

where μ is the direction cosine with respect to the x-axis, μ_s is the cosine of the scattering angle, Σ_s is the macroscopic scattering cross section, and $q(x', E, \mu)$ is the angular source calculated from $\phi(x', E', \vec{\Omega}')$ by integration over all incoming angles.

The P_1 approximation to the angular flux is introduced within the angular source integral. This application of the P_1 approximation is much less restrictive than that used in P_1 theory (which leads to diffusion theory) as the spatial attenuation of the source neutrons is treated exactly using the integral transport equation; the calculated angular flux can have any angular distribution. Substituting the P_1 approximation for the angular flux and describing the strict correlation between energy transfer and scattering angle by the Dirac delta function, Eq. (6) becomes:

$$q(x', E, \mu) = \frac{1}{4\pi}\int_{\Omega'}\int_{E'}\Sigma_s(E' \rightarrow E)\delta[\mu_s - \mu_s(E', E)]\phi(x', E')dE'd\Omega' + \frac{1}{4\pi}\int_{\Omega'}\int_{E'}\Sigma_s(E' \rightarrow E)\delta[\mu_s - \mu_s(E', E)](3\vec{\Omega}' \cdot \vec{J}(x', E'))dE'd\Omega'. \tag{7}$$

Evaluation of the double integrals in Eq. (7) is simplified by expressing $\vec{\Omega}'$ and $\vec{J}(x', E')$ in a coordinate system around $\vec{\Omega}$ (the direction of the scattered neutron, where μ is the direction cosine of $\vec{\Omega}$). This is advantageous because $\vec{\Omega}$ is constant in both double integrals. The evaluation of these two angular integrals yields the angular scattering source as:

$$q(x', E, \mu) = \frac{1}{2}\int_{E'}\Sigma_s(E' \rightarrow E)\left\{\phi(x', E') + 3\mu\mu_s(E', E)J(x', E')\right\}dE'. \tag{8}$$

For the analysis of the transitional resonance streaming effects, the distinction of forward and backward motion is of primary interest since the streaming effects are to be quantified. It was shown in Eqs. (5) that streaming leads to opposite effects on the forward and backward directed angular fluxes. Thus, the angular flux is integrated over a cone in the forward (+) and backward (-) directions to isolate the streaming effects. The simplest case as presented here is the integration over each half-space; however, the equations have also been derived for angular cones. The flux obtained from the half-space angular integration has the form:

$$\phi^+(x, E) = \int_0^1\int_{-\infty}^{x}\frac{e^{-\frac{\Sigma_t(E)(x - x')}{\mu}}}{\mu}q(x', E, \mu)dx'd\mu\ . \tag{9}$$

Insertion of the angular source, Eq. (8), and performing the μ integration yields:

$$\phi^+(x, E) = \frac{1}{2}\int_{-\infty}^{x}\left\{E_1\left[\Sigma_t(E)(x - x')\right]\int_{E'}\Sigma_s(E' \rightarrow E)\phi(x', E')dE' + 3E_2\left[\Sigma_t(E)(x - x')\right]\int_{E'}\Sigma_s(E' \rightarrow E)\mu_s(E', E)J(x', E')dE'\right\}dx'. \tag{10a}$$

$$\phi^-(x, E) = \frac{1}{2}\int_{x}^{\infty}\left\{E_1\left[\Sigma_t(E)(x' - x)\right]\int_{E'}\Sigma_s(E' \rightarrow E)\phi(x', E')dE' - 3E_2\left[\Sigma_t(E)(x' - x)\right]\int_{E'}\Sigma_s(E' \rightarrow E)\mu_s(E', E)J(x', E')dE'\right\}dx'. \tag{10b}$$

The first term in Eqs. (10a,b) is the commonly used scattering source term for integral transport codes such as RABANL. This flux term comes from an isotropic source approximation (J = 0 in the source integral). The second term contains the angular-energy correlation of the scattering source accounting for the forward bias (as expressed in the current) of the incoming neutrons. Neutrons with a strong forward bias (large neutron current) which scatter through small scattering angles (large scattering angle cosine) will tend to remain forward biased. Thus, this second term models the spatial-energy coupling caused by the correlation between energy loss and scattering angle.

The neutron current needed for the second integral in these equations, is calculated in a similar manner. Fick's Law does not apply near a resonance since there is obviously not free diffusion at the resonance energy. Therefore, an independent current (not based on an approximation of the angular flux) is calculated by utilizing the integral transport equation for the one-dimensional angular flux and the definition of the neutron current:

$$J(x, E) = \int_{-1}^{1} \mu \phi(x, E, \mu) d\mu. \tag{11}$$

The current is similarly divided into forward and backward partial spaces and the corresponding angular integrations are performed.

III.B Application of Theory

Equations (10) and the corresponding current component equations form a coupled set of integral equations since the half-space components are used to calculate $\phi(x, E)$ and $J(x, E)$ which are in the integrands. A numerical solution method was developed which minimizes the approximations in calculating the integrands and integrals.

As discussed previously, this model will address the problem of an isolated resonance. Thus, an "input" flux and current above the resonance are needed to calculate the scattering source into the resonance. For this model, the assumed spectrum above the resonance will be 1/E for the flux and current. The spatial variation is approximated by the multigroup flux and current from a slab diffusion calculation. The multigroup spatial solution is discretized at several spatial mesh points (typically 75 points); thus, the integral transport equations are solved at these same mesh points.

As another aspect of the isolation assumption, only the resonance isotope will have cross section variations in the region of interest. This assumption should be quite accurate within narrow resonances as addressed in this paper; in the analysis of the transitional effects of broad scattering resonances this assumption would not be valid. In addition, constant cross sections for all isotopes are assumed in the energy interval above the resonance from which neutrons are scattered into the resonance. Thus, the scattering source above the resonance has the same 1/E variation as the flux.

The numerical solution of these equations is simplified because downscattering leads to energy integral domains that include only energies above (and including) the point of interest. Therefore, a "stepping-down in energy" procedure is used to treat the energy variable. With appropriate discretization of the spatial and energy variables, the equations can be solved at the highest energy point. Using this result the equations can then be solved at the next energy point, and the entire energy range is covered in a similar manner.

The energy integration of Eqs. (10) is carried out analytically above the resonance range; the s-wave scattering kernel was used for this integration. Within the resonance, the trapezoidal rule is used. This requires that a further assumption be made between the energy point of interest and the last energy point only. Very fine energy discretization (more than 100 points per resonance) is used and a constant flux value for this small energy range is assumed.

The spatial integration of Eqs. (10) can be performed analytically if a linear spatial variation of the source between mesh points is assumed. This integration of the exponential integrals is based on the work in Ref. 5 and is detailed in Ref. 4.

In summary, a method has been developed to calculate the spectra for an isolated resonance as a function of space. This model makes no assumption regarding the angular distribution of the flux within the resonance and treats the spatial attenuation exactly with integral transport theory. The P_1 approximation of the angular flux is utilized in calculating the scattering source. The model is applied to an isolated resonance, assuming a smooth energy dependence of the spatial multigroup flux solution above the resonance. Only the cross section variations of the individual resonance are modeled. Thus, for a given spatial multigroup flux distribution, material composition, and resonance, one can calculate the appropriate transitional resonance spectra.

IV. ANALYSIS OF AN ISOLATED ACTINIDE RESONANCE

IV.A Specific Application of Model

The motivation for developing this capability was the presence of calculational errors in fast reactor blanket predictions. Specifically, the C/E drop-off at the Purdue Fast Breeder Blanket Facility (FBBF) was addressed. Thus, this investigation considers a blanket of the FBBF composition.

The material composition is based on the FBBF model discussed in Ref. 1. The spatial flux profile was generated by performing a one-dimensional "FBBF-like" slab calculation using the FBBF mid-plane number densities with an extended blanket to allow analysis of a somewhat deeper penetration. The blanket multigroup flux profile should provide an adequate model of the spatial dependence of the scattering source despite the calculational errors identified previously: the C/E drop-off to .8 is small compared to the overall flux decrease by about a factor of 150.

The neutrons entering the blanket are described by a surface flux. The surface flux is calculated using a simplified version of Eqs. (10); the simplification is the neglect of the neutron current in the scattering integral in generating the surface flux.

To assess transitional resonance spectra, it is useful to look at the spectral effects for a specific actinide resonance. Since the interference effects are governed by the resonance scattering width, one would expect the largest effects to be for resonances with a large Γ_n. In addition, the fast reactor spectrum is concentrated at higher energies well above 100 eV. Based on these considerations, a U-238 resonance at 2.2 keV was chosen for this comparison ($\Gamma_n = 590$ meV, $\Gamma_\gamma = 24.3$ meV, resonance parameters taken from Ref. 6). Doppler broadened resonance cross sections are generated in RESITT using the GENRESM program developed by Mo.[7,8] This resonance is located in group 29 of the 50 group energy structure; thus, the group 29 spatial flux profile is used.

A computer code, RESITT, was written utilizing the described integral transport methodology and integration methods. Resonance parameters, potential cross sections, blanket composition, a spatial flux profile, and a surface flux (from the simplified calculation) are all input to the RESITT code.

IV.B Infinite Medium Flux Comparisons

As discussed in Section II, the NR approximation assumes a scattering source that is constant in energy across the resonance. However, the integral transport analysis will calculate an energy dependent source, Eq. (2). An infinite medium spectrum for the energy dependent scattering source was calculated in order to allow a separate consideration of energy and spatial effects.

The upper half of Figure 1 shows several comparisons:

- First, the NR-spectrum, $\phi^{NR}(E)$, is compared with the correct (energy dependent source) infinite medium spectrum, $\phi^{\infty}(E)$, indicating a considerable difference. The increase in the interference peak of the infinite medium spectrum compared to the NR approximation is caused by resonance scattering. The high scattering cross section in the resonance peak leads to a larger source than the NR approximation which uses only the potential cross section. Similarly, the slight dip well below the resonance is caused by the low scattering cross section of the interference dip leading to a depressed U-238 scattering source.
- Furthermore, Fig. 1 shows the surprising fact that the transitory flux-spectrum, $\phi(x,E)$, plotted here at 40 cm blanket penetration, agrees closely with the infinite medium spectrum, $\phi^{\infty}(E)$, both calculated with integral transport theory.
- Less surprising is the fact that the spectrum for the cone complement, $\phi_{cc}(E)$, which pertains to the angular domain with a forward and backward-directed 90° cone subtracted, agrees equally well with $\phi^{\infty}(E)$. This is expected because the sideward integrations in a slab geometry are similar to the infinite medium integrations since the slab is infinite in the transverse direction. The ramifications of these agreements are discussed below.

IV.C Spatial Neutron Flux Comparisons

The RESITT code was applied to generate the space dependent resonance spectra throughout the extended blanket region. A comparison of the integral transport theory predictions at 40 cm blanket penetration to the infinite medium approximation is shown in the lower half of Fig. 1. where ϕ_c^+ and ϕ_c^- refer to 90° cones in the forward and backward directions respectively. The infinite medium spectrum agrees well with the spatial results near the converter/blanket interface. However, within the blanket the forward directed components, $\phi^+(x,E)$ and $\phi_c^+(x,E)$, show a large flux increase outside the resonance peak and especially in the interference dip. Opposite effects are observed in the backward directed components. The deviations of the components from the infinite medium spectrum compound with penetration.

The effects in the forward directed flux components correspond to the expected streaming effects. The relative importance of the low cross section energies increases because of the slower attenuation rate. However, these same streaming effects lead to the opposite trend in the backward directed flux components since the source has the opposite behavior (if the source is decreasing in the positive direction, then it is increasing in the negative direction). Therefore, when one adds the forward and backward directed components to form the total flux, the streaming effects cancel out to an amazing degree as shown in the upper half of Fig. 1.

This numerical result can be understood analytically in the simplified treatment expressed in Eqs. (5). The first derivative term of the source expansion cancels out when the forward and backward fluxes are added and only the much smaller second derivative term remains.

The spatial changes in the spectra observed in Fig. 1 are more extreme for the cone components than for ϕ^+ and ϕ^-. This difference is attributed to the fact that by integrating over the entire angular half-space the streaming effects are "diluted" by large contributions from angles near the "infinite" direction; thus, the cone results lead to a better isolation of the one-dimensional transition effects.

If the spectra $\phi^+(x, E)$ and $\phi^-(x, E)$ of Fig. 1 were used to generate two separate sets of group constants, the forward-directed scattering group constants would be smaller than those calculated with $\phi^\infty(E)$ since they give a higher weight to small $\Sigma_s(E)$ than does $\phi^\infty(E)$. Furthermore, the group constants would decrease with penetration, allowing increasingly more transmission. Conversely, the backward-directed scattering group constant would increase, inhibiting backward scattering. Thus, the directional effects will complement each other to yield higher neutron transmission in blanket predictions. However, when the components are combined to form a weighting flux, the observed differences cancel ($\phi(x,E)$ agrees very closely with $\phi^\infty(x,E)$) and the transitional spectra effects are lost. Therefore, increasing the accuracy of flux weighting-spectra for the generation of group constants has no effect in increasing blanket transmission; direction dependent (flux component weighted) group constants need to be used instead.

V. PRELIMINARY EVALUATION OF DIRECTION DEPENDENT GROUP CONSTANTS

The purpose of this evaluation is to assess the effect of spatially dependent resonance spectra on group constants. The transitional resonance effects are accounted for by utilizing the transitory resonance spectra in place of the NR approximation in group constant generation. This significantly complicates the group constant computation because the transitional spectra are both space and direction dependent (unlike the infinite medium NR approximation).

The forward and backward half-space flux components can be used to generate two different group constant sets. These group constants would yield increased blanket transmission predictions when applied in an S_2 transport calculation utilizing direction dependent group constants. Alternatively, the two 90° cone spectra, $\phi_c^+(x,E)$ and $\phi_c^-(x,E)$, give a more pronounced description of the angular dependence of the spectra. They can be used to generate two group constant sets which are combined with a set applying the infinite medium self-shielding approximation as $\phi^\infty(E)$ agrees well with $\phi_{cc}(E)$. These three group constant sets, used in an S_4 (or higher S_n) calculation should yield even more accurate fluxes in transition problems since the large portion of the angular flux, $\phi_{cc}(x,E)$, which agrees well with the infinite medium spectrum has been split off. It should be noted that all current transport theory codes use direction-independent, i.e. "diffusion theory group constants;" thus, application of direction dependent group constants will require significant modification of existing methods.

This preliminary investigation analyzes the transitory scattering self-shielding factors for the forward and backward directed half-space flux components, $\phi^+(x, E)$ and $\phi^-(x, E)$. This analysis is further simplified by applying the 2.2 keV resonance results throughout the energy group which exaggerates the effect on the group constant as this resonance has one of the largest Γ_n-values in this group. Because of this simplification, the analysis is merely an illustration of transitional group constant effects for the blanket problem.

The scattering self-shielding factor is defined as:

$$f_s(x) = \frac{\sigma_{sG}(x)}{\sigma_{sG,id}(x)}, \qquad (12)$$

i.e. as ratio of group averaged self-shielded and infinite dilution cross sections. The corresponding self-shielded resonance reaction rate is calculated by utilizing the integral transport solution within the resonance and applying the narrow resonance approximation to the (far out) wings as detailed in Ref. 4. Thus, the self-shielding factor at x, $f_s(x)$ is generated by using the transitional weighting spectrum at x in the calculation of the self-shielded resonance reaction rate.

Figure 2 presents f-factors based on flux, infinite medium, backward directed flux component, and forward directed flux component spectral weighting; the percentage difference compared to the NR f-factor is plotted. The integral transport infinite medium f-factor is slightly larger than the NR f-factor. The flux-weighted group constants decrease with penetration; however, even for this extreme case the f-factor deviates only about 3% from the infinite medium results across the blanket. The special interface effects seen in the flux weighted group constants in Fig. 2 have been treated earlier by Mo, Ref. 7.

If the group constants are generated using $\phi^+(x, E)$ and $\phi^-(x, E)$ instead of the flux, they are now representative of S_2 direction dependent group constants. Large deviations are seen for these group constants in Fig. 2; this finding is consistent with the large spectral deviations seen in these quantities. The forward directed scattering f-factor increases by about 5% across the first 20 cm of the blanket; the backward directed f-factor decreases by about 5% over the same distance leading to a 10% difference between the two directional group constants. These differences increase continuously throughout the transition region to about a 20% difference between the two directions 60 cm into the blanket. As described previously, these deviations are complementary, both lead to more transmission; thus, this preliminary investigation indicates that the differences between forward and backward directed group constants are large enough to have a significant effect on the flux calculation.

SUMMARY

An integral transport theory for analyzing spatially dependent neutron spectra within isolated resonances was developed in this paper. This transitional resonance analysis is designed to replace the NR approximation as the source of within resonance weighting spectra for group constant generation. The attenuation effects are treated rigorously using the integral transport equation with no assumption regarding the angular flux distribution. A P_1-representation for the angular distribution of the flux was used only within the scattering source integral.

Transitional effects on the within-resonance spectrum are illustrated for a ^{238}U resonance near 2.2 keV. For proper comparison the NR spectrum is replaced by the transport theoretical infinite medium spectrum. The transitional effects lead to large changes in the forward and backward directed resonance spectra. However, these effects cancel when the components are combined to form the flux weighting spectrum which agrees very closely with the infinite medium spectrum. Thus, an accurate description of streaming in transition regions requires refined group constant generation using different weighting spectra for forward and backward directed neutrons. The integral transport formalism presented here allows the calculation of the forward and backward directed spectra in a transition region; therefore, it is the first step toward the generation of the required direction dependent group constants.

The differences in forward and backward directed group constants will increase blanket neutron transmission. The decrease in forward scattering group constants will allow more penetration (streaming); conversely, increased scattering group constants will inhibit backward motion. Therefore, the directional effects complement each other to yield higher blanket predictions.

In conclusion, selective results indicate that transitional resonance effects are of a magnitude that they could account for the discrepancies observed in worldwide blanket and other neutron transmission results. Thus, the onset of the deep penetration problem in the blanket region can likely be eliminated by utilizing refined direction dependent group constants. It was shown that a reduction of the transmission discrepancies is not possible by using diffusion theory group constants, even if they are based on rigorous within-group flux weighting spectra. An appropriate description of the spatial attenuation requires the treatment of direction dependent group constants.

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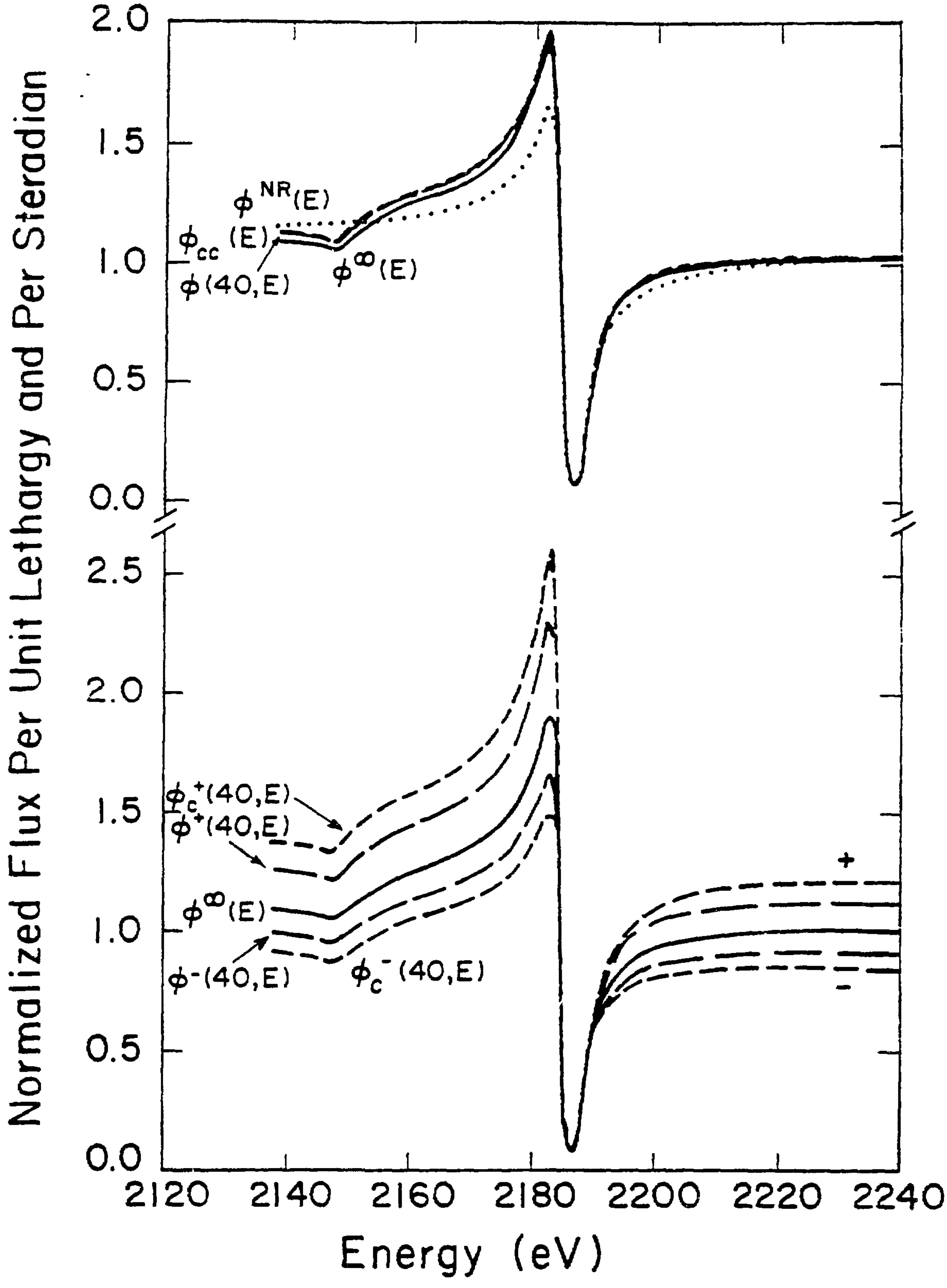

Fig. 1. Comparison of within resonance weighting spectra.

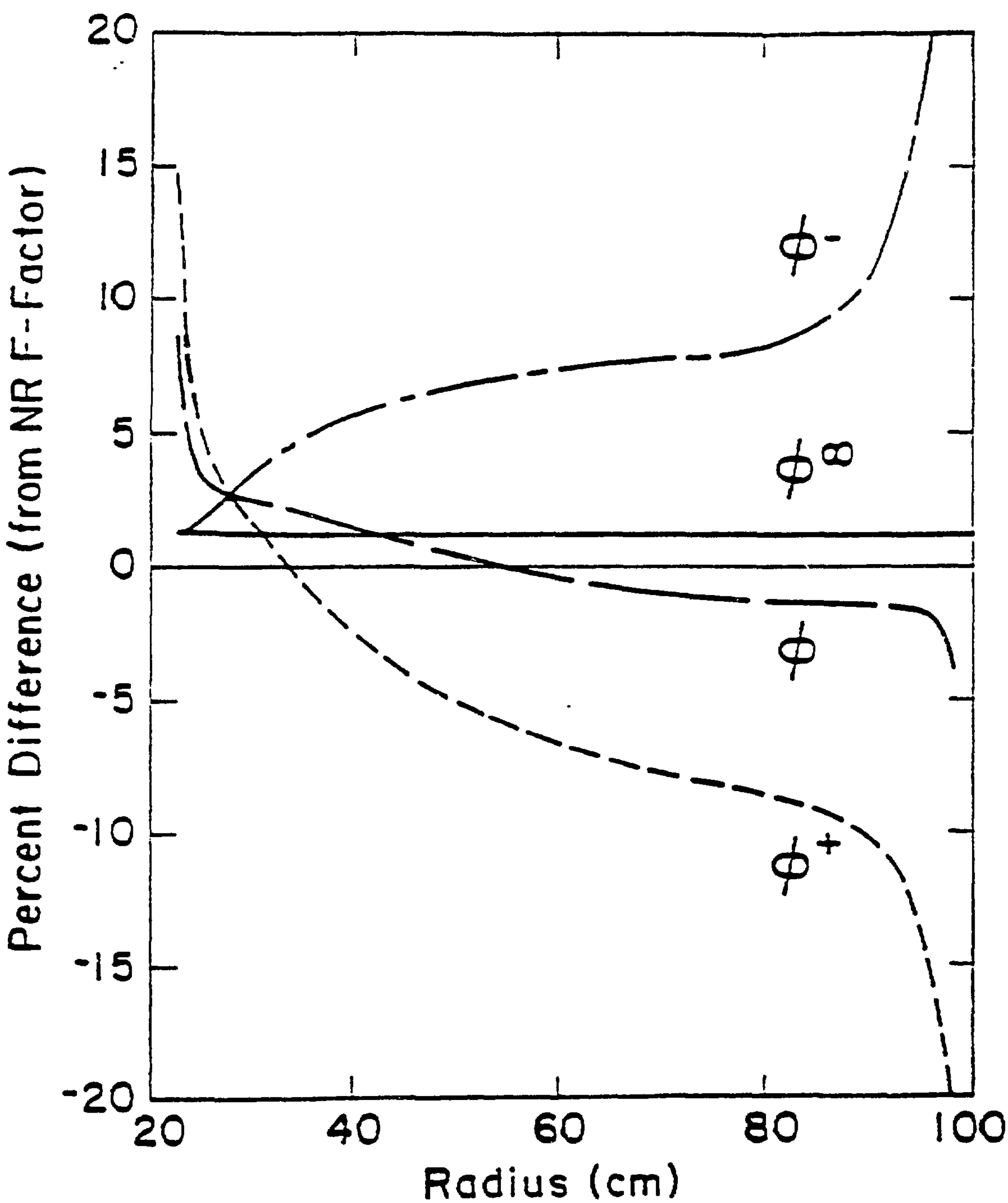

Fig. 2. Spatial variations of U-238 scattering self-shielding factors, $f_s(x)$, calculated with different weighting spectra: $\phi^\infty(E)$, $\phi(x,E)$, $\phi^+(x,E)_g$ and $\phi^-(x,E)$. Presented are the percent difference of $f_s(x)$ and the space independent NR-result.